Experimental Estimation of Modeling Errors in Dynamic Systems

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This paper presents a method for estimating the errors of a linear, time-invariant model of a dynamic system. The estimation is based on system inputs and outputs recorded during an experiment. The errors are expressed as tolerances of the model parameters. These tolerances are such that a simulation of the dynamic system in which the linear model parameters are allowed to vary within their estimated tolerance intervals can reproduce the response recorded during the experiment. The method is based on linear programming, which guarantees that the identified errors are the smallest possible that can reproduce the experimental data. The model error information is useful for failure detection and isolation, system simulation, and control law design. The method is illustrated using a flight-dynamics example in conjunction with the reachable measurement intervals failure detection method for systems with modeling errors.

Nomenclature

A = system matrix
B = input matrix
B_I = input matrix equal to the identity matrix
C = output matrix
D = direct transmission matrix
F = vector of cost coefficients for linear-programming problem
G = vector of upper bounds of parameter tolerances
H = Pontryagin's function
L = vector of Lagrange multipliers
m = number of system outputs
N = number of time steps in experimental record
n = number of system states
p = discrete-time index corresponding to current time
P = vector of constants for least-squares problem
R = vector of constants for linear-programming problem
r = number of system inputs
s = sensitivity of a state to variations in an entry of A
T = sampling period
t = time
t_0 = current time
u = system input
W = matrix of weights for linear-programming problem
x = system state
y = system output
z = sensitivity of a state to variations in an entry of B

Subscripts

c = constant correction
d = discrete
e = deviation from nominal
n = nominal
t = tolerance
v = time-varying correction

Introduction

MANY system identification methods for linear time-invariant systems have been developed and analyzed over the last three decades.[1] These methods cover a broad range of applications and generate models with various desirable statistical properties. Although these methods can deal effectively with the uncertainty introduced by process and measurement noise, there is little they can do to deal with the nonlinearities and the time-varying characteristics that are present to some extent in all systems. Because of the fixed structure of these models, they can at best linearize the nonlinearities and represent time-varying parameters by their average values. These differences between the systems and their models are called modeling errors.

These approximations usually do not cause problems if the identified models are intended for control law design, because the models contain the information required to meet the stability and response criteria of the controlled system. Furthermore, feedback is usually used to reduce the effect of nonlinearities and parameter variations on the response of the closed-loop system.

The situation is different, however, if the models are used for failure detection and isolation. Model-based failure detection and isolation in dynamic systems are based on comparison of the measured response with the estimated response obtained using a model of the system. The difference between the two responses is used to detect the presence of failures. Failure detection would be straightforward if exact system models were available and systems were noise-free. Exact modeling of real systems, however, is impossible, and noise affects all real systems. Several effective methods are available for dealing with process and sensor noise in failure detection systems,[2] all of which rely on the fact that window averaging reduces the effect of noise while not reducing the failure signature. This paper concentrates on the more difficult issue of modeling errors, which are the primary obstacle to failure detection in mechanical and thermofluid systems.

Modeling errors cause differences between the estimated and the measured responses even in the absence of failures. These errors limit the sensitivity of the failure detection algorithms because they mandate the use of decision thresholds in order to prevent frequent false alarms. The thresholds, while preventing false alarms caused by modeling errors, also prevent the detection of failures that cause output deviations smaller than the thresholds. Therefore, the sensitivity of failure detection systems is limited by the modeling errors, no matter how small they are. From the failure detection point of view, modeling errors can never be negligible.

The best failure detection threshold[3] for systems with modeling errors has the smallest possible size that guarantees no false alarms. This best threshold is a time-varying quantity because, in addition to modeling errors, it also depends on the

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state and inputs of the system. The best threshold makes the detection of the smallest reliably detectable failures possible. A threshold smaller than the best does not prevent false alarms, whereas one larger than the best is too conservative and unnecessarily misses small failures. Our reachable measurement intervals (RMI)[3] failure detection and isolation method for systems with modeling errors uses the best threshold and, therefore, can detect the smallest theoretically detectable failures. Since the best threshold is a function of the modeling errors, the accurate computation of the threshold can only be accomplished if the modeling errors are known accurately. Inaccurate knowledge of the modeling errors must be compensated for by an empirically determined threshold that is larger than the best if frequent false alarms are to be prevented.

Ideally, one would like to use a very accurate, nonlinear, time-varying and high-order model for failure detection. Such a model would have small errors, allowing the detection of small failures. However, such an approach is not practical because of the difficulties in determining the model structure and parameters, and because of the computational difficulties involved in using such models in real-time failure detection algorithms. Therefore, most failure detection algorithms are based on linear, time-invariant models. This paper concentrates on the estimation of errors of such models.

Modeling errors can be classified as structured or unstructured.[4] Structured errors can be represented as uncertainties of parameters of a time-domain model. Unstructured errors can be represented as uncertainties of the transfer function matrix of the model. The frequency-domain representation of modeling errors is useful in robust control system design.[4] The experimental estimation of these errors based on comparison of computed and measured frequency responses is relatively simple. This representation, however, is not detailed enough for accurate and fast failure detection and isolation. It cannot be used for computation of the best threshold previously mentioned, which requires a time-domain model of the errors.

We use the following representation of the monitored system:

$$\dot{x} = [A_n + A_v + A_c]x + [B_n + B_v + B_c]u \tag{1a}$$

$$y = [C_n + C_v + C_c]x + [D_n + D_v + D_c]u \tag{1b}$$

The system has n states, r inputs, and m outputs. It is assumed to be noise-free at this stage. The effect of noise will be added to the analysis in a later section. Matrices A_n, B_n, C_n, and D_n in Eqs. (1) are the nominal model. Matrices A_v, B_v, C_v, and D_v represent time-varying parameter corrections that are required to account for the time-varying errors caused by the inaccuracy of the nominal model. Each term of these matrices is only known to lie in a tolerance interval defined by a lower and an upper bound. The tolerance intervals, assumed symmetric with respect to the nominal parameters, are defined by matrices A_t, B_t, C_t, and D_t. The terms of these matrices are half the widths of the corresponding tolerance intervals. For example, a term of the matrix A_v is bounded by $-a_{tij} \le a_{vij} \le a_{tij}$.

Matrices A_c, B_c, C_c, and D_c represent a time-invariant correction to the nominal model. This correction is useful in cases when the nominal model parameters are biased with respect to the parameters of the monitored system. The corrected nominal model described by $A_n + A_c$, $B_n + A_c$, $C_n + C_c$, and $D_n + D_c$ makes the assumption that the tolerances are symmetric with respect to the nominal parameters more accurate. Note that the constant correction is not required if the nominal model has been identified using an identification method that produces unbiased parameter estimates.

This paper assumes that a linear, time-invariant, time-domain model of the dynamic system is available, having been derived analytically or identified experimentally using any one of the many known methods. The contribution of this paper is a method for experimentally estimating the errors of this model. The errors are expressed as tolerance intervals within which the parameters of the model must be allowed to vary during a simulation in order to reproduce the response of the monitored system during an experiment. The method is based on linear programming, and can be implemented using any general-purpose Simplex program. Two versions of the methods have been developed. One is suitable for parameters that can vary faster than the speed of response of the monitored system, and the other for parameters that vary more slowly than the response of the system. The two versions complement the two RMI failure detection methods for systems with modeling errors we have developed, one for rapidly varying parameters[3,5] and the other for slowly varying parameters.[3,6]

The paper first develops expressions for sensitivities of system measurements to parameter variations. These expressions are then used to derive the main result, the method for estimating the parameter tolerances required to account for the time-varying effects caused by the errors of the nominal model. A flight-dynamics example is then used to illustrate the method.

Formulation of the Problem

General Model of the Dynamic System

The parameters of the nominal model of a dynamic system can be determined analytically if the model is available in symbolic form, or experimentally using any one of the many available parameter identification methods. This model represents the average behavior of the system. It cannot represent real hardware exactly, because all physical systems include nonlinearities, unmodeled dynamics, randomly varying parameters, and other effects that cannot be represented accurately by models of realistically low size and complexity.

The nominal model is usually adequate for control law design because the unmodeled effects, which are relatively small, are reduced even further by feedback. This model, however, is not sufficient for accurate failure detection and isolation because the small effects it neglects are the main issue in failure detection algorithms. They set the limits on failure detectability in terms of the size of the smallest reliably detectable failure.[3] Failures smaller than the limit cannot be detected without causing frequent false alarms, because their effect is comparable in size to the effect of the modeling errors.

Sensitivity of Measurements to Parameter Variations

The estimation of the modeling errors is based on an experiment in which the inputs and the outputs of the monitored system are recorded at N points with sampling period T. The model of the system is

$$\dot{x} = A_n x + B_n u + A_e x + B_e u \tag{2a}$$

$$y = C_n x + D_n u + C_e x + D_e u \tag{2b}$$

The first two terms on the right-hand sides of Eqs. (2) represent the nominal model. The last two terms represent the deviation from the nominal model. Since it is assumed that the system is stable, there is a time pT during which the response of the system to its initial conditions settles to a negligible value. Therefore, it is possible to express the current state of the system as a sum of the contributions of its inputs over the last pT seconds. Applying this approach to the last two terms of the discrete equivalent of Eqs. (2) yields

$$x(p) = x_n(p) + \sum_{q=1}^{p} A_{nd}^{p-q} B_{Id}[A_e x_n(q-1) + B_e u(q-1)] \tag{3}$$

where x_n is computed with the nominal model, time index $q = p$ designates present time, A_{nd} is the discrete system matrix obtained via the zero-order-hold approximation, and B_{Id} is the discrete equivalent of an input matrix equal to the identity matrix. B_{Id} is required to account for the continuous correction terms in the discrete summation. Note that in Eq. (3) the state x in the product $A_e x$ from Eqs. (2) has been replaced by x_n, where its value is estimated with the nominal model. The use of this approximation is necessary since in the general case the state vector of the monitored system is not measured. The approximation introduces only small errors if the nominal model of the system is reasonably accurate.

It is now possible to evaluate the sensitivity of the states to the terms of matrices A_e and B_e. Let the sensitivity of state x_k to a_{eij}, a term of the matrix A_e, be $s_{kij} = \partial x_k / \partial a_{eij}$. By differentiating Eq. (3) it is

$$s_{kij} = \sum_{q=1}^{p} (A_{nd}^{p-q} B_{Id})_{k,i} x_{nj}(q-1) \tag{4}$$

where x_{nj} is nominal state number j, and $(\ )_{k,i}$ designates the term in row k and column i of a matrix. Similarly, the sensitivity to an uncertain term of the matrix B_e is defined as $z_{kil} = \partial x_k / \partial b_{eil}$. By differentiating Eq. (3) it is

$$z_{kil} = \sum_{q=1}^{p} (A_{nd}^{p-q} B_{Id})_{k,i} u_l(q-1) \tag{5}$$

where u_l is input number l.

It is now possible to rewrite Eq. (3) for state number k in terms of the sensitivities as

$$x_k = x_{nk} + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{eij} s_{kij} + \sum_{i=1}^{n}\sum_{l=1}^{n} b_{eil} z_{kil} \tag{6}$$

Note that since Eq. (3) is linear in A_e and B_e, Eq. (6) does not include any approximations with respect to Eq. (3).

Consider initially only one output of the system. From Eqs. (2) and (6) its value is

$$y = \sum_{k=1}^{n} (c_{nk} + c_{ek}) \left[x_{nk} + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{eij} s_{kij} + \sum_{i=1}^{n}\sum_{l=1}^{r} b_{eil} z_{kil} \right] + \sum_{w=1}^{r} (d_{nw} + d_{ew}) u_w \tag{7}$$

The expression for the deviation of the output from its nominal value is obtained by subtracting the nominal system equations from Eq. (7) and by neglecting second-order correction terms. It yields

$$y - y_n = \sum_{k=1}^{n} \left[\sum_{i=1}^{n}\sum_{j=1}^{n} a_{eij} c_{nk} s_{kij} + \sum_{i=1}^{n}\sum_{l=1}^{r} b_{eil} c_{nk} z_{kil} \right] + \sum_{k=1}^{n} c_{ek} x_{nk} + \sum_{w=1}^{r} d_{ew} u_w \tag{8}$$

Equation (8) can be used to compute the output deviations because of the known *parameter deviations* a_e, b_e, c_e, and d_e. The equation will be used in the following section to derive the main result of this paper, a method for estimating the *parameter tolerances* that are required to explain the measured output deviations.

Estimation of Parameter Tolerances

This section describes the main result of this paper, a method for estimating the parameter tolerances that are required in order to account for the time-varying part of the errors of the nominal model. There are two types of parameter variations: those that can vary faster than the speed of response of the system, and those that vary more slowly. The rapidly varying parameters can change significantly during time pT, the settling time of the system. The slowly varying parameters cannot change significantly during that time.

Rapidly Varying Parameters

The goal is a set of parameter tolerances that define intervals for the parameter values. These intervals are such that a simulation of the system with its parameters varying within the intervals can reproduce the measurements recorded during the experiment. We are after the tightest possible tolerances that are sufficient to reproduce every measured output. The tightest possible tolerances are associated with those time histories of parameter variations that produce the largest differences between the measured outputs and the nominal outputs. The computation of these time histories is closely related to the RMI algorithm.[3] The system, initially assumed to have only one output, is described by

$$\dot{x} = A_n x + B_n u + A_v x + B_v u \tag{9a}$$

$$y = c_n x + d_n u + c_v x + d_v u \tag{9b}$$

The terms of A_v, B_v, c_v, and d_v represent the parameter variations from their nominal values. Their absolute values are limited by the parameter tolerances A_t, B_t, c_t, and d_t, which are to be determined. Similar to Eq. (3), the state of the system is given by

$$x(p) = x_n(p) + \sum_{q=1}^{p} A_{nd}^{p-q} B_{Id} [A_v(q-1) x_n(q-1) + B_v(q-1) u(q-1)] \tag{10}$$

Equation (10) represents a family of responses that differ in the time-varying terms A_v and B_v. One specific response yields the state that produces the largest possible value of $c_n x$, the contribution of the varying parameters to the system output. This response can be found by maximizing $c_n x$ via application of the Maximum Principle[3,7] to the system in the time window pT, starting in the past and ending at the present time. The performance index for maximization of $c_n x$ is

$$PI = c_n x(t_0) \tag{11}$$

where t_0 is the present time, corresponding to time index p. The Pontryagin H function is given by

$$H = L(t)^T [A_n x + B_n u + A_v x + B_v u] \tag{12}$$

where L is the vector of Lagrange multipliers.

The differential equations for the Lagrange multipliers are obtained by partial differentiation of $-H$ from Eq. (12) with respect to the states x, which yields

$$\dot{L}(t) = -A_n^T L(t) - A_v^T L(t) \tag{13}$$

As explained in more detail in Ref. 3, in systems with a reasonably accurate nominal model, Eq. (13) can be approximated by

$$\dot{L}(t) = -A_n^T L(t) \tag{14}$$

without causing significant errors in the final solution. The values of L can be obtained by simulating Eq. (14) backward in time from the terminal conditions

$$L(t_0) = c_n^T \tag{15}$$

With the values of L known, the Maximum Principle can be used to determine the maximizing values of A_v and B_v. They are those values that maximize H in Eq. (12). An entry of A_v is positive if the sign of the Lx term multiplying it is positive, and it is negative otherwise. The value of a term of B_v is determined similarly by the sign of Lu. Note that the Maximum Principle selects only the extreme values of the uncertain terms. Therefore, the absolute values of the entries of A_v and

B_v are always equal to the tolerances defined by A_t and B_t. The time-varying nature of the optimal solution is only in the varying polarity of the variable terms. This property of the optimal solution makes the estimation of the tolerances possible. Equation (10) can be rewritten to match this special form of the optimal solution as

$$x(p) = x_n(p) + \sum_{q=1}^{p} A_{nd}^{p-q} B_{Id}[A_{t\sigma} x_n(q-1) + B_{t\sigma} u(q-1)] \tag{16a}$$

$$a_{t\sigma ij} = \sigma_{aij} a_{tij} \tag{16b}$$

$$b_{t\sigma il} = \sigma_{bil} b_{til} \tag{16c}$$

$$\sigma_{aij} = \text{sgn}[L_i(q-1)x_{nj}(q-1)] \tag{16d}$$

$$\sigma_{bil} = \text{sgn}[L_i(q-1)u_l(q-1)] \tag{16e}$$

By differentiating Eqs. (16), the sensitivities of the states to parameter variations are

$$s_{kij} = \sum_{q=1}^{p} (A_{nd}^{p-q} B_{Id})_{k,i} \sigma_{aij} x_{nj}(q-1) \tag{17}$$

$$z_{kil} = \sum_{q=1}^{p} (A_{nd}^{p-q} B_{Id})_{k,i} \sigma_{bil} u_l(q-1) \tag{18}$$

We now have all the tools required to formulate the mathematical solution of the parameter tolerance estimation problem. The solution is based on Eq. (8), modified to handle *tolerances* of rapidly varying parameters rather than parameter variations. Two modifications are required. First, all the constant terms of the equation are replaced by their absolute values to be consistent with the definition of tolerances as nonnegative quantities. Second, the equality is replaced by an inequality to reflect the fact that the tolerances are the upper limits of the parameter variations. The final form is

$$|y - y_n| \le \sum_{k=1}^{n} \left[\sum_{i=1}^{n} \sum_{j=1}^{n} a_{tij} |c_{nk} s_{kij}| + \sum_{i=1}^{n} \sum_{l=1}^{r} b_{til} |c_{nk} z_{kil}| \right] + \sum_{k=1}^{n} c_{tk} |x_{nk}| + \sum_{w=1}^{r} d_{tw} |u_w| \tag{19}$$

Consider a column vector M_t, which consists of all the entries of the tolerance matrices and vectors, and a row vector R, which includes the other terms of Eq. (19) in such order that

$$RM_t \ge |y - y_n| \tag{20}$$

is equivalent to Eq. (19). If the duration of the experiment is NT, N equations such as Eq. (20) can be constructed. If there are m outputs, m equations can be constructed at every time step. Therefore, there are Nm equations such as Eq. (20). Since this inequality can always be satisfied by sufficiently large tolerances, the problem must be solved as a constrained optimization:

$$\text{minimize} \quad FM_t \tag{21a}$$

$$\text{subject to} \quad RM_t \ge |y - y_n| \quad (Nm \text{ inequalities}) \tag{21b}$$

where F is a row vector of nonnegative cost coefficients, R is a row vector of constants, and M_t is a column vector of parameter tolerances to be determined. The inequality in Eq. (21b) is a compact representation of Eq. (19).

The optimization problem in Eqs. (21) is the linear-programming problem that can be solved with any standard Simplex routine. A useful initial set of weights F are the inverse magnitudes of the corresponding nominal parameters, which will produce a solution in which the tolerances are approximately equal percentages of the nominal parameter values. It is possible to set upper bounds on the tolerances by adding Q more inequalities to Eq. (21b) (Q is the number of unknown tolerances). These inequalities are

$$-IM_t \ge -G \tag{22}$$

where G is a column vector of the upper bounds, and I is the identity matrix of dimension Q.

The entire optimization problem of determining the tolerances of the rapidly varying uncertain parameters is described by Eqs. (16–19), (21), and (22). The final result is the vector M_t of parameter tolerances.

Slowly Varying Parameters

Although it is possible to formulate the tolerance estimation problem for slowly varying parameters using the Maximum Principle, as done for the rapidly varying parameters, at this point it can be done more compactly by referring to the previous sections.

Since the slowly varying parameters do not change significantly in the processing window of the algorithm, the sensitivities of states to parameter variations are given by Eqs. (4) and (5). Except for the different sensitivities, the solution is exactly as the solution for the rapidly varying parameters. The entire optimization problem is summarized in Eqs. (4), (5), (19), (21), and (22).

Algorithm Use and Implementation

The absolute values of the states' sensitivities to changes in parameters are always larger for rapidly varying parameters than for slowly varying parameters. This difference is so because the rapidly varying parameters have more freedom to change so as to drive the system outputs away from their nominal values. Consequently, since the parameter tolerances and the sensitivities appear as products in Eq. (19), the tolerances under the rapidly varying parameter assumption are always smaller than those under the slowly varying parameter assumption. The difference can be as low as a few percent in overdamped systems or as high as hundreds of percent in oscillatory systems.

The tolerances in both cases are derived under the assumption that the uncertain parameters take on only the extreme values allowed by their tolerances. This assumption results in the tightest possible parameter tolerances that can reproduce the experimental data. Since the parameters of real systems do not vary in such ideal fashion, the computed tolerances must be viewed as estimates of the lower limits of the actual parameter variations. These lower limits are exactly the parameter variations required by the RMI algorithm to reproduce the experimental data, because RMI also uses only the extreme values of the parameters.

An engineer using the developed modeling error estimation method must combine it with his knowledge of the monitored system in order to produce the best possible results. If the system parameters are known to be rapidly varying within their tolerances during normal operation, such as the cornering coefficients of tires at high speed, the sensitivities for rapidly varying parameters should be used. If, on the other hand, the parameters are known to be slowly varying, such as temperature-dependent friction in a servoactuator, the slowly varying sensitivities should be used. Note that the two types of sensitivities can be mixed in one system as long as the use of the estimated tolerances is consistent with their derivation.

Although even the first run of the linear-programming algorithm will produce a set of parameter tolerances that can reproduce the experiment, it is desirable to derive tolerances that agree with any a priori model accuracy information, if it is available. This can be accomplished in an iterative process in which the weights F in Eq. (21) and the upper bounds G in Eq. (22) are varied so as to produce a set of tolerances that reflect the relative modeling accuracy of the system components.

The modeling error estimation method assumes that the differences between the model and the system are entirely caused by parameter errors. Naturally, it performs best in such situations. However, useful results also can be generated when the differences result from nonlinearities and unmodeled dynamics. For example, a nonlinear function of a state $f(x)$ will be automatically approximated by $(f \pm f_t)x$, where $\pm f_t$ is the estimated tolerance interval. This approximation is quite accurate for functions such as nonlinear gain, dead-band, or saturation. Obviously, if during the error estimation experiment a saturating component never reaches saturation, the nonlinear behavior will not be reflected in the estimated parameter tolerances.

The model in Eq. (2) does not have provisions for accurate modeling of severe nonlinearities, such as products of states, or unmodeled high-order dynamics. Therefore, the estimation method will assign wider tolerance intervals to those parameters that can affect the outputs in a way that resembles the effect of the nonlinearities or the neglected dynamics. This approach will still produce useful results if the modeling error experiment includes all of the maneuvers the system will perform during normal operation. However, if during normal operation a nonlinear phenomenon that did not occur during the experiment occurs, the estimated parameter tolerances will not be able to account for it.

The error estimation algorithm has been derived for a noise-free situation. It is used in this form in cases when the effect of process and sensor noise on the system outputs is much smaller than the output estimation errors caused by the modeling errors. The linear-programming algorithm will automatically estimate parameter tolerances that are slightly larger than in the noise-free case, thus compensating for the neglected noise. If the effect of noise is not small, this approach is not appropriate, because additive noise cannot be represented accurately by parameter variations. This situation can be handled by appropriately modifying the constraints, Eq. (19), as described next.

Equation (19) states that the parameter tolerances must be large enough to explain the deviation of a measurement from the output computed with the nominal model. In most cases process and sensor noise affect the measurement and, depending on their polarity, they either increase or decrease the estimation error $|y - y_n|$. A constraint for a time-step when the noise increases the error will force the solution of the linear-programming problem toward larger parameter tolerances, which is undesirable. This problem can be avoided by reducing the left-hand side of Eq. (19) at every point by the largest value the noise is likely to contribute to the estimation error (but not below zero), such as three times the standard deviation in the Gaussian case. Note that modifying the estimation error at times when the noise decreases it does not cause problems, because Eq. (19) for such points will be easily satisfied and will not affect the solutions of the linear-programming problem.

It is sometimes possible for a single estimation error to be affected by noise of a magnitude that is well above that predicted by the probability density function used to model the noise. Such a point, even if it is the only one out of thousands used in the modeling error estimation, will force the solution toward very high tolerances. It is advisable to reject that point from the data set before attempting to solve the linear problem. This approach is consistent with the noise handling approach used in the RMI algorithm,[3] which will not issue an alarm if only a single measurement exceeds the threshold. The selection of the points to be rejected can be based on Eq. (19) in the following way. All of the tolerance terms in the equation are replaced by the absolute nominal values of the corresponding parameters, and the ratio of the left-hand side of the equation to the right-hand side is evaluated for all the points in the data set. An irregular point is one with a ratio much larger than its neighbors, indicating that it will require much larger parameter tolerances than the other points to explain the estimation error.

The system inputs used during the parameter tolerance estimation experiment should resemble the inputs that excite the system during normal operation. Since the modeling error estimation method and the RMI failure detection algorithm use the same principle to handle the errors, every system maneuver included in the estimation experiment is guaranteed correct processing by the RMI algorithm when the system is being monitored during normal operation.

Note that there is no need to use all of the recorded experimental data as input to the linear-programming algorithm. Initially, one can skip points at which the left-hand side of Eq. (19) is small, and represent groups of adjacent points with similar constants by just one representative point. Once initial estimates of the parameter tolerances are available, new data points can be skipped if they do not violate Eq. (19). Thus, even many hours of experimental data can be reduced to only several thousand useful data points.

The execution time of the Simplex algorithm for the example problem described below was 32 s on a 20 MHz 80386/387 computer, when all 7500 points were considered. The algorithm was coded in structured Fortran. When the number of data points was reduced to 2500, using the techniques described above, the execution time decreased to 9 s. The tolerances estimated with 2500 data points were less than 4% off the values estimated with 7500 data points. Note that the parameter sensitivities can be computed recursively as the data is being collected.[3]

The execution speed of the Simplex algorithm is sufficient to make estimation of the tolerances during the system's mission possible. In an aircraft battle damage detection application, for example, the first few minutes of flight can be used to generate the model error information that can, subsequently, be used by the RMI algorithm to detect battle damage. It is also possible to use the algorithm in an adaptive scheme in which the modeling errors are re-evaluated every few minutes.

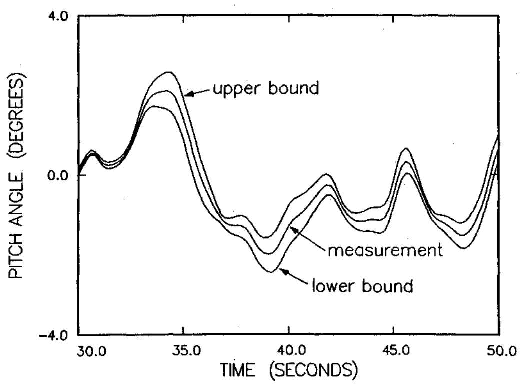

Fig. 1 Pitch angle measurement and reachable intervals computed with parameter tolerances set at $\pm 5\%$ of the nominal parameter values.

Constant Model Correction

If the nominal model on which the estimation of the tolerances is based has not been determined by parameter identification, it may be biased with respect to the monitored system parameters. This also happens if the model is derived analytically, or if the identification is performed using a different unit than the specific one used for estimating the tolerances. It is desirable to minimize the parameter unbiased before estimating the parameter tolerances, because an inbiased nominal model requires smaller tolerances to reproduce an experiment. Therefore, it makes the detection of smaller failures possible.

The parameter bias can be reduced by application of least-squares to Eq. (8) (with the subscript c replacing the subscript e). Consider a column vector M_c, which consists of all the entries of the constant correction matrices A_c, B_c, C_c, and D_c and a row vector P, which includes the other terms of the equation in such order that

$$PM_c = y - y_n \tag{23}$$

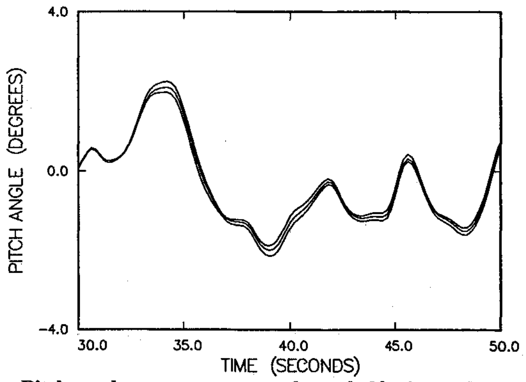

Fig. 2 Pitch angle measurement and reachable intervals computed with identified parameter tolerances but without constant parameter corrections.

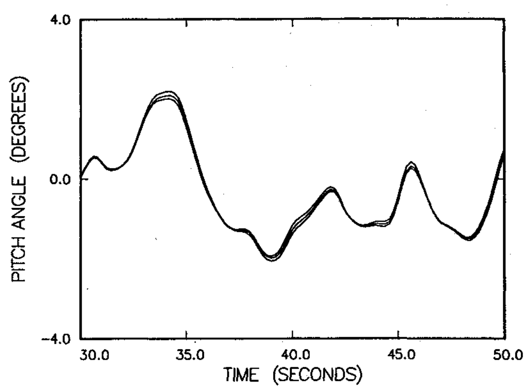

Fig. 3 Pitch angle measurement and reachable intervals computed with identified parameter tolerances and with constant parameter corrections.

is equivalent to Eq. (8). A general-purpose least-squares routine can be applied to Eq. (23) to obtain the solution M_c. The corrected nominal model is defined by matrices $A_n + A_c$, $B_n + B_c$, $C_n + C_c$, and $D_n + D_c$.

In order to avoid unreasonably large constant correction terms, it is useful to add the sum of their weighted squared values to the cost function of the least-squares optimization. It can be achieved by adding to the Nm equations in Eq. (23) Q more equations, where Q is the total number of correction terms. These equations have the form

$$gWM_c = 0 \tag{24}$$

where W is a diagonal matrix of dimension Q, in which the diagonal terms are the positive weights for the corresponding correction terms, and g is a positive constant. A good initial set of weights is the inverse absolute values of the nominal parameters. The least-squares solution is only loosely dependent on the constant g. There is a wide range of values of g, found by trial and error, for which the solution remains almost constant.

Example

The modeling error estimation method is illustrated on a fifth-order model of the longitudinal dynamics of the F16 aircraft.[5] The aircraft was simulated on a VAX computer, with its seven uncertain aerodynamic parameters allowed to deviate by up to 5% from the nominal values in a slow, random fashion. The pitch angle, pitch rate, and angle of attack measurements produced in the simulation were used for the estimation of parameter tolerances. The simulation lasted 500 s, and the sampling time was 0.2 s. Thus, the number of equations in Eqs. (21b) and (23) was $Nm = (500/0.2)3 = 7500$.

The pitch angle reachable measurement intervals[3] computed with the RMI algorithm for slowly varying parameters using the nominal 5% parameter tolerances are shown in Fig. 1. The RMI algorithm guarantees that the pitch angle measurement will be within the reachable measurement interval, which is the smallest possible failure detection threshold that guarantees no false alarms caused by the modeling errors, as long as the aircraft parameters are within their tolerances. A measurement outside of its interval indicates a failure.

Since during the 500 s simulation only a small subset of the possible parameter variations occurred, the intervals in Fig. 1 are unnecessarily wide. Figure 2 shows the intervals as computed with the parameter tolerances that were estimated based on the 500 s simulation. These intervals are about 3 times narrower than those in Fig. 1. The constant model parameter corrections have not been applied in this case. Figure 3 shows the intervals with the constant model corrections applied before estimating the parameter tolerances. These intervals are 5–8 times narrower than the nominal intervals in Fig. 1. The constant corrections reduced the width of the reachable intervals further because they removed the parameter biases introduced during the simulation. With an unbiased model, smaller parameter tolerances were sufficient to account for the time-varying part of the modeling errors.

The example illustrates the importance of the parameter tolerance estimation method. The estimated tolerances are the tightest possible that are sufficient for reproducing all the experimental data. The tight tolerances allow the RMI algorithm to use narrower reachable measurement intervals, which in turn make the detection of smaller failures possible. The size of the smallest detectable failure in this example has been reduced by a factor of 8.

Conclusions

A method for experimental estimation of the errors of linear, time-invariant models of dynamic systems has been developed. The method expresses the estimated errors as tolerances of the nominal model parameters. These tolerances are such that a simulation of the system with its parameters varying within their tolerances can reproduce the experimentally measured response of the system. The estimation method is based on linear programming and can be implemented using standard Simplex algorithms.

The parameter tolerances can be used as data for the reachable measurement intervals (RMI) failure detection method for systems with modeling errors, which we developed previously. This failure detection method combined with the modeling error estimation method forms an integrated tool for analysis, design, and implementation of failure detection systems in complex applications. Laboratory experiments in which we first estimate the parameter tolerances of an experimental system and then use them for detecting failures are now in progress.

In addition to its use in conjunction with the RMI failure detection method, the developed tolerance estimation method is also useful for determining the accuracy of dynamic models used for system simulation and control.

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